Maximum Likelihood Estimation of Lift and Drag from Dynamic Aircraft Maneuvers

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A maximum likelihood estimation method for obtaining lift and drag characteistics from dynamic flight maneuvers was investigated. This paper describes the method and compares the estimates of lift and drag obtained by using the method with estimates obtained from wind-tunnel tests and from established methods for obtaining estimates from flight data. In general, the lift and drag coefficients extracted from dynamic flight maneuvers by the maximum likelihood estimation technique were in good agreement with the estimates obtained from the wind-tunnel tests and the other methods. When maneuvers that met the requirements of both flight methods were analyzed, the results of each method were nearly the same. The maximum likelihood estimation technique showed promise in terms of estimating lift and drag characteristics from dynamic flight maneuvers. Further studies should be made to assess the best mathematical model and the most desirable type of dynamic maneuver to get the highest quality results from this technique.

Nomenclature

a_N = normal acceleration, g
a_X = longitudinal acceleration, g
C_D = coefficient of drag
C_{D_0} = coefficient of drag at zero angle of attack
$\bar{C}_{D_0}$ = coefficient of drag at zero lift
C_L = coefficient of lift
C_m = coefficient of pitching moment
C_X = coefficient of longitudinal force
C_Z = coefficient of normal force
c = vector of unknown coefficients
D = force parallel to relative wind
D_1 = weighting matrix
E = augmented state matrix
F = augmented control matrix
F_n = net thrust, N
$f(\cdot)$ = generalized state function
G = augmented observation matrix
g = acceleration due to gravity, m/sec^2
$g(\cdot)$ = generalized observation function
H = augmented control matrix for observation
I_Y = moment of inertia about lateral axis, kg-m^2
i = index variable
J = cost functional
k_1,k_2,k_3 = constant values of elevator deflection, deg
L = force perpendicular to relative wind
$\mathfrak{M}$ = normalized pitching moment, rad/sec^2
M_r = drag-rise Mach number ratio
m = aircraft mass, kg
N = number of time points
q = pitch rate, deg/sec or rad/sec
$\bar{q}$ = dynamic pressure, N/m^2
S = reference area, m^2
s = augmented state vector
T = total observation time, sec
t = incremental time, sec
u = control vector
V = velocity, m/sec
W = aircraft weight, N
w = augmented control vector
$\mathfrak{X}$ = normalized force along X axis, m/sec^2
X = X-body axis
x = state vector
y = observation vector
$\mathfrak{Z}$ = normalized force along Z axis, m/sec^2
Z = Z-body axis
z = measurement vector
α = angle of attack, deg or rad
δ_e = elevator deflection, deg or rad
ζ = angle between X axis and thrust axis, deg
η = measurement noise vector
ν = measurement bias vector
θ = pitch angle, deg or rad
φ = roll angle, deg or rad

Subscripts

i = time index
$q,V,\alpha,\alpha^2,\alpha\delta_e,\delta_e,\delta_e{}^2$ = partial derivative of subscripted variable with respect to subscript
trim = trimmed value
0 = nominal or constant value

Superscripts

$(\ )^*$ = matrix transpose
$(\dot{\ })$ = derivative with respect to time

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Introduction

An important task in any flight test program is to estimate airplane lift and drag characteristics throughout the flight envelope on the basis of aircraft parameter measurements. The actual drag of a vehicle is particularly important in that the vehicle's overall performance is determined largely by the magnitude of the drag. Current methods obtain estimates of drag from steady-state conditions or from pushover/pullup or windup-turn maneuvers. These maneuvers must be performed precisely to avoid significant changes in flight conditions and to insure that pitching motion and dynamic aeroelastic effects during the maneuver are insignificant. These methods of determining lift and drag characteristics are referred to as steady-state or quasisteady-state techniques and are described in Ref. 1.

In any method, it is necessary to have an accurate estimate of the vehicle's thrust in order to determine drag. Without estimates of thrust, the most complete information that can be obtained is the net forces normal and parallel to the thrust axis. The net force parallel to the thrust axis is also of interest for applications where only the estimates of excess thrust are required.

Experience has shown that is is frequently impossible to perform quasisteady-state maneuvers without changes in velocity, Mach number, or pitching motion that are unacceptable in terms of the assumptions made for quasisteady-state maneuver analysis.

It would therefore be of interest to be able to obtain aircraft lift and drag characteristics from flight with less restrictive maneuvers. Obtaining lift and drag from dynamic maneuvers has the advantage of placing less restriction on the pitching motion during the maneuver and of providing additional information, such as trim drag, instrument biases, and aircraft pitching-moment characteristics.

This paper evaluates a maximum likelihood estimation method of estimating lift and drag from dynamic flight maneuvers. The estimation method evaluated applies equally well to the determination of excess thrust. Perhaps its greatest advantage over existing methods is that it can determine excess thrust for a flexible maneuvering aircraft. The equations that describe the aircraft's dynamic motion and the algorithm for obtaining maximum likelihood estimates are developed. The trimmed lift and drag coefficients computed from the maximum likelihood estimates of the unknown coefficients of the dynamic equations are compared with the coefficients estimated with the wind tunnel and with the methods described in Ref. 1. The method is completely developed and all of the data are presented in Ref. 2.

Method of Analysis

The problem considered is: given a set of flight time histories of an aircraft's response variables, find the values of some unknown parameters in the system equations that best represent the actual aircraft response. An intuitive mathematical approach to this problem is to minimize the difference between the flight response and the response computed from the system equations. This difference could be defined for each response variable as the integral of the error squared. These responses could then be multiplied by weighting factors proportional to the relative confidence in each signal and summed to obtain the total weighted response error. This defines an integral squared error criterion.

A mathematically more precise probabilistic formulation can be made. For each possible estimate of the unknown parameters, a probability that the aircraft response time histories attain values near the observed values can be defined. The estimates should be chosen so that this probability is maximized. This process is the maximum likelihood formulation of the problem.

Maximum Likelihood Estimation

When the basic assumptions for the method are met, maximum likelihood estimation[3] has many desirable characteristics; for example, it yields asymptotically unbiased and consistent estimates. If measurement noise is assumed to be Gaussian, white, stationary, and uncorrelated, this formulation is equivalent to the response error formulation described earlier where the weightings used are the inverse of the measurement noise covariance matrix.

To describe maximum likelihood estimation mathematically, it is first necessary to define the equations of motion that describe the aircraft phenomenon of interest. In general, these equations can be written as follows:

$$\dot{x}(t)=f(x,u,t) \tag{1}$$

$$y(t)=g(x,u,t) \tag{2}$$

$$z(t)=y(t)+\eta(t) \tag{3}$$

where

x = state vector
u = control vector
y = computed observation vector
z = measured observation vector
η = noise vector

The cost functional can be defined as the integral squared error criterion as follows:

$$J=\frac{1}{T}\int_0^T [z(t)-y(t)]^*D_1[z(t)-y(t)]\,\mathrm{d}t \tag{4}$$

or approximated in the discrete case:

$$J=\frac{1}{N-1}\sum_{i=1}^{N}(z_i-y_i)^*D_1(z_i-y_i) \tag{5}$$

where D_1 is the symmetric, nonnegative definite weighting matrix, i is the time index, and N is the number of time points. The cost functional J can also be called the index of performance or the fit error.

To obtain maximum likelihood estimates for a given set of flight responses, it is necessary to minimize the cost functional J. Since $z(t)$ is fixed for given flight responses, J must be minimized by selecting the $y(t)$ that minimizes J where $y(t)$ is subject to the constraints of Eqs. (1) and (2).

Development of Mathematical Model

To define this procedure, a precise definition of $f(x,u,t)$ and $g(x,u,t)$ must be made. The definitions of $f(x,u,t)$ and $g(x,u,t)$ (in other words, the mathematical model) depend on the types of information it is desirable to extract from the flight responses. Therefore, it is necessary to define the problem precisely before defining the mathematical model. The problem considered in this paper is the extraction of lift and drag information from dynamic flight maneuvers where all the motion is in the longitudinal modes. The axis system, along with the other quantities needed to describe the vehicle, is shown in Fig. 1. If the aircraft motions are small enough, the mathematical model can be defined as linear. Then the lift and drag can be defined in a piecewise linear manner over a wide range of lift coefficients or angles of attack. However, where the motion is large, as in the standard pushover/pullup maneuver for conventional aerodynamic configurations, theoretical considerations dictate that the drag increase approximately as the square of the lift. Therefore, a reasonable model for defining drag could be written as follows:

$$C_D=\bar{C}_{D_0}+C_{D_{C_{L^2}}}C_L{}^2 \tag{6}$$

The lift can be approximated fairly well as a first-order Taylor-series expansion of angle of attack and elevator deflection. The following equation results from this approximation:

$$C_L=C_{L_0}+C_{L_\alpha}\alpha+C_{L_{\delta_e}}\delta_e \tag{7}$$

The substitution of Eq. (6) into Eq. (7) results in the following equation:

$$C_D=\bar{C}_{D_0}+C_{D_{C_{L^2}}}(C_{L_0}+C_{L_\alpha}\alpha+C_{L_{\delta_e}}\delta_e)^2 \tag{8}$$

Upon expansion and redefinition of terms, Eq. (8) can be written as follows:

$$C_D = C_{D_0} + C_{D_\alpha}\alpha + C_{D_{\delta_e}}\delta_e + C_{D_\alpha}\alpha^2 + C_{D_{\alpha\delta_e}}\alpha\delta_e + C_{D_{\delta_{e2}}}\delta_e^{\,2} \tag{9}$$

The instrumentation system of the aircraft remains fixed during flight, so all the measurements are referenced to a fixed-axis system called the body-axis system. Lift and drag forces are defined with respect to an axis system that is referenced to the relative wind, and consequently the axis system varies during flight. The lift and drag coefficients, C_L and C_D, can be written with respect to the normal and longitudinal coefficients, C_Z and C_X, as follows:

$$C_L = -C_Z\cos\alpha + C_X\sin\alpha \tag{10}$$

and

$$C_D = -C_X\cos\alpha - C_Z\sin\alpha \tag{11}$$

Since the contributions to C_D are due to C_X and C_Z, it is desirable to write the equations used to describe C_X and C_Z in the same form as used to define C_D in Eq. (9). The equations are as follows:

$$C_X = C_{X_0} + C_{X_\alpha}\alpha + C_{X_{\delta_e}}\delta_e + C_{X_{\alpha 2}}\alpha^2 + C_{X_{\alpha\delta_e}}\alpha\delta_e + C_{X_{\delta_{e2}}}\delta_e^{\,2} \tag{12}$$

$$C_Z = C_{Z_0} + C_{Z_\alpha}\alpha + C_{Z_{\delta_e}}\delta_e + C_{Z_{\alpha 2}}\alpha^2 + C_{Z_{\alpha\delta_e}}\alpha\delta_e + C_{Z_{\delta_{e2}}}\delta_e^{\,2} \tag{13}$$

Equations (12) and (13) describe the coefficients of normal and longitudinal force as functions of α and δ_e that can be used to define the aircraft mathematical model.

The differential equations of motion should then be written to include the power-series expansions given in Eqs. (12) and (13) as well as the other effects due to the dynamic behavior of the aircraft. These additional effects can be specified in terms of the aircraft state and control variables. The states that are required are pitch rate q, angle of attack α, total velocity V, and pitch angle θ. The control variables are elevator deflection δ_e and constants to account for biases. The observations (response variables) provided by the aircraft instrumentation system are q, α, V, θ, pitch acceleration $\dot{q}$, normal acceleration at the center of gravity a_N, and longitudinal acceleration at the center of gravity a_X. The state vector x, the control vector u, and the observation vector y, can be written as follows:

$$x = \begin{bmatrix} q \\ \alpha \\ V \\ \theta \end{bmatrix} \qquad u = \begin{bmatrix} \delta_e \\ 1 \end{bmatrix} \qquad y = \begin{bmatrix} q \\ \alpha \\ V \\ \theta \\ \dot{q} \\ a_N \\ a_X \end{bmatrix}$$

After the states and observations have been selected, a suitable nonlinear model should be defined to describe them. Since the nonlinear effects defined by Eqs. (12) and (13) are expected to be functions of both angle of attack and elevator deflection, they must be included in the model. Keeping the number of nonlinear terms as small as possible increases the probability of attaining the minimization and also of obtaining meaningful estimates. Thus, the differential equations of motion below were chosen. They allow for known nonlinearities but do not include higher-order nonlinearities, which would greatly increase the complexity of the model. Notationally it is most convenient to write the equations in terms of dimensional quantities (normalized pitching moment $\mathfrak{M}$ and the normalized aerodynamic forces $\mathfrak{X}$ and $\mathfrak{Z}$). These quantities have the following relationships to the dimensionless coefficients:

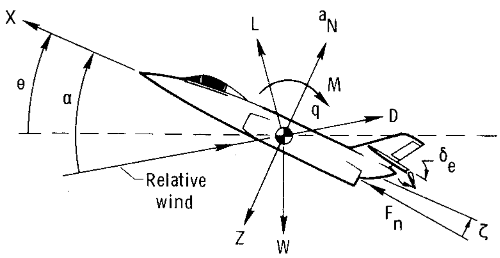

Fig. 1 Axis system and sign convention used for aircraft equations.

$$\mathfrak{M} = C_m\bar{q}S/I_Y \tag{14}$$

$$\mathfrak{X} = C_X\bar{q}S/V \tag{15}$$

$$\mathfrak{Z} = C_Z\bar{q}S/(mV) \tag{16}$$

Thus,

$$\dot{q} = \mathfrak{M}_V V + \mathfrak{M}_q q + \mathfrak{M}_\alpha\alpha + \mathfrak{M}_{\alpha 2}\alpha^2 + \mathfrak{M}_{\delta_e}\delta_e + \mathfrak{M}_{\delta_{e2}}\delta_e^{\,2} + \mathfrak{M}_{\alpha\delta_e}\alpha\delta_e + \mathfrak{M}_0 \tag{17}$$

$$\dot{\alpha} = \mathfrak{Z}_V V + q + \mathfrak{Z}_\alpha\alpha + \mathfrak{Z}_{\alpha 2}\alpha^2 + \mathfrak{Z}_{\delta_e}\delta_e + \mathfrak{Z}_{\delta_{e2}}\delta_e^{\,2} + \mathfrak{Z}_{\alpha\delta_e}\alpha\delta_e + \mathfrak{Z}_0 + (g/V)\cos\varphi\cos\theta_0\cos\theta^{1} - (g/V)\cos\varphi\sin\theta_0\sin\theta^{\theta} \tag{18}$$

$$\dot{V} = \mathfrak{X}_V V + \mathfrak{X}_\alpha\alpha + \mathfrak{X}_{\alpha 2}\alpha^2 + \mathfrak{X}_{\delta_e}\delta_e + \mathfrak{X}_{\delta_{e2}}\delta_e^{\,2} + \mathfrak{X}_{\alpha\delta_e}\alpha\delta_e + \mathfrak{X}_0 + (F_n/m) - g\sin\theta_0\cos\theta^{1} - g\cos\theta_0\sin\theta^{\theta} \tag{19}$$

$$\dot{\theta} = q \tag{20}$$

$$a_N = -(V/g)(\mathfrak{Z}_V V + \mathfrak{Z}_\alpha\alpha + \mathfrak{Z}_{\alpha 2}\alpha^2 + \mathfrak{Z}_{\delta_e}\delta_e + \mathfrak{Z}_{\delta_{e2}}\delta_e^{\,2} + \mathfrak{Z}_{\alpha\delta_e}\alpha\delta_e + \mathfrak{Z}_0) \tag{21}$$

$$a_X = (1/g)(\mathfrak{X}_V V + \mathfrak{X}_\alpha\alpha + \mathfrak{X}_{\alpha 2}\alpha^2 + \mathfrak{X}_{\delta_e}\delta_e + \mathfrak{X}_{\delta_{e2}}\delta_e^{\,2} + \mathfrak{X}_{\alpha\delta_e}\alpha\delta_e + \mathfrak{X}_0 + F_n/m) \tag{22}$$

These equations make the assumptions that the thrust axis and the $\mathfrak{X}$-body axis are coincident and that the accelerometers are referenced to the center of gravity. These equations can be written in more general form if these two assumptions are invalid by adding the proper terms in Eqs. (17-19, 21, and 22). The lift and drag coefficients can be calculated with Eqs. (10-13) by using the coefficients estimated with Eqs. (14-22). Equations (17-22) can be written in a more convenient form by using the manipulations described later.

If an augmented state vector s and an augmented control vector w are defined as follows:

$$s = \begin{bmatrix} x \\ \cdots \\ \alpha^2 \\ \alpha\delta_e \end{bmatrix} \qquad w = \begin{bmatrix} \delta_e \\ \delta_e^{\,2} \\ 1 \\ 1 \end{bmatrix}$$

the following state and observation equations result:

$$\dot{x} = Es + Fw \tag{23}$$

$$y = Gs + Hw + \nu \tag{24}$$

where the matrices E, F, G, and H are defined by Eqs. (17-22). The vector ν is to account for each of the biases in the observations.

The equations for a_N and a_X are written with respect to the center of gravity location for simplicity. Higher-order nonlinearities, such as functions of α^3, α^4, and $\alpha^2\delta_e^2$, can be included in the foregoing formulations at the expense of increased complexity. One way to determine whether a more complex system is necessary is to take any a priori estimates, usually wind-tunnel estimates, of the lift and drag and to see whether they can be adequately represented by the model and, if not, which additional terms are required. It is unwise to include variables that are not essential, because the resulting estimates of lift and drag may become less meaningful even though y may more nearly approximate z.

After explicit expressions for x and y have been defined by using Eqs. (23) and (24), the procedure for minimizing J [Eq. (4)] by adjusting y can be stated clearly. The computer observation vector y is completely defined by the unknown elements of the matrices E, F, G, and H and the measurement bias vector ν. The value of J can be minimized by selecting the unknown elements that result in y as close to z (in the mean square sense) as possible. The minimization can be more conveniently stated by defining a vector of unknowns c that contains all the unknown elements of E, F, G, H, and ν. The maximum likelihood estimates will then result if J in Eq. (4) is minimized with respect to c. The procedure for the minimization is described in Ref. 2.

Analysis Procedure

To complete the definition of the analysis procedure once the analysis technique and mathematical model have been defined, the input data must be specified. The input data required can be inferred from the variables in Eqs. (17-22). That is, measurements of the observations q, α, V, θ, $\dot{q}$, a_N, and a_X and of the control input δ_e are used in the following analysis. The maximum likelihood estimation method does not require all of these observations, but they are used here since they are available for the data set to be analyzed. The operational procedure used for the following analysis of flight data is similar to that described in Ref. 4.

Results and Discussion

After a model and a method for obtaining maximum likelihood estimates of lift and drag from dynamic flight maneuvers have been selected, the method should be tested with high-quality flight data. The data should not contain pitching motions large enough to cause significant aircraft deformations or unsteady aerodynamic effects. For this study, high-quality data were available from a set of pushover/pullup maneuvers made by a fighter aircraft. High-quality wind-tunnel estimates and estimates obtained from flight data by using the steady-state and quasisteady-state techniques described in Ref. 1 were also available. The pitching motion in these data was lower than would be expected to cause structural deformation or unsteady aerodynamic effects. The data are from an actual aircraft, although the data have been modified so that the vehicle is not readily identifiable because exact drag values for a particular aircraft can be controversial. The aircraft is relatively simple in that the propulsion system does not interact significantly with the nonpropulsive aerodynamic characteristics of the aircraft. The data include maneuvers in the transonic region. Relaxing the restriction on pitching motion allows the maneuvers to be performed more quickly and with small variations in dynamic pressure and Mach number, which is important in the transonic regime. The maneuvers that were available were all performed for the purpose of quasisteady-state analysis. Thus, none of the maneuvers was performed very rapidly, although the pitching motion during some of the maneuvers was too great to permit acceptable quasisteady-state analysis.

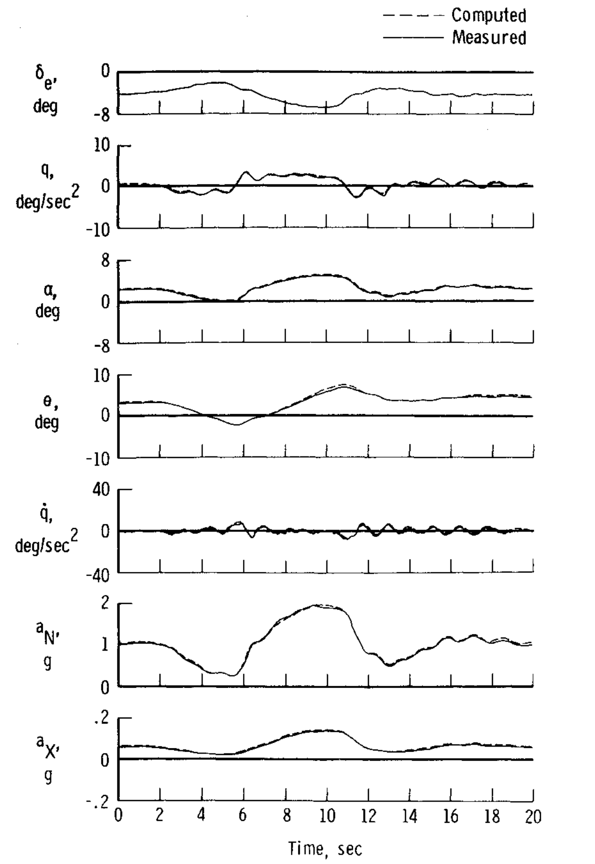

Fig. 2 Comparison of measured and computed responses for relatively fast pushover/pullup maneuver.

Some of the pushover/pullup maneuvers were performed more rapidly than the others, and one of these maneuvers was used as the test case for the algorithm being discussed. The results of that analysis are shown in Fig. 2. The computed responses are based on the estimates of the converged maximum likelihood estimation algorithm. The agreement of the fit of the computed with the measured responses is excellent, indicating that the model defined by Eqs. (23) and (24) is at least sufficiently complex to represent the aircraft adequately for a relatively rapid maneuver. This maneuver produced the best fit obtained of all the maneuvers analyzed in this study. The fit may be best because the maneuver was performed rapidly, resulting in flight conditions that may have been more nearly constant.

Twenty-one pushover/pullup maneuvers were performed at an altitude of approximately 13,500 m and at Reynolds numbers ranging from $8\text{-}12 \times 10^6$ (based on the mean aerodynamic chord) over the Mach number range of interest. Of the 21 maneuvers, the analysis of 14 maneuvers was deemed to be successful. An example of a good fit for a more typical maneuver is shown in Fig. 3. This fit, although not the best obtained, is considered to be good overall, with some small errors evident in the match for θ.

The remaining seven of the 21 maneuvers provided unacceptable results. During one maneuver the Mach number variation during the maneuver was too large. For three maneuvers, the fit between the measured and the computed data based on the converged maximum likelihood estimates was unsatisfactory. For the remaining three maneuvers, the maximum likelihood estimation technique did not converge to a stable solution. Therefore, suitable maneuvers where no

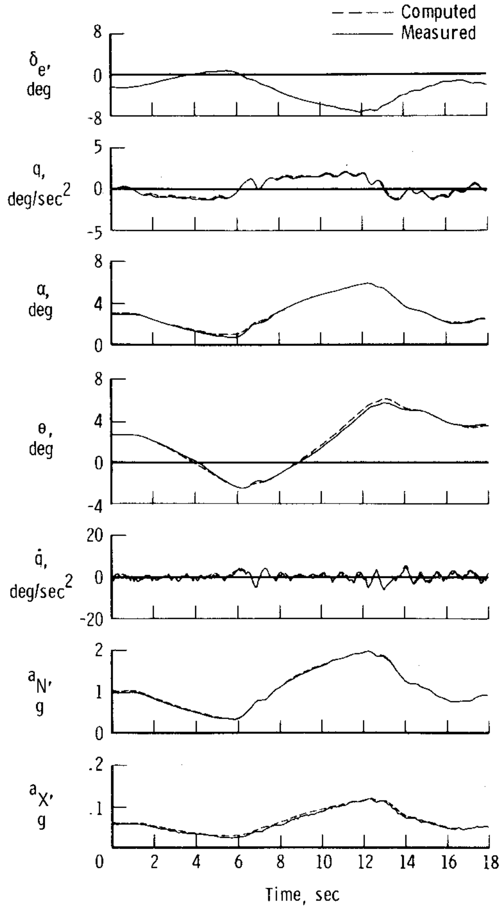

Fig. 3 An example of good agreement between measured and computed responses for pushover/pullup maneuver.

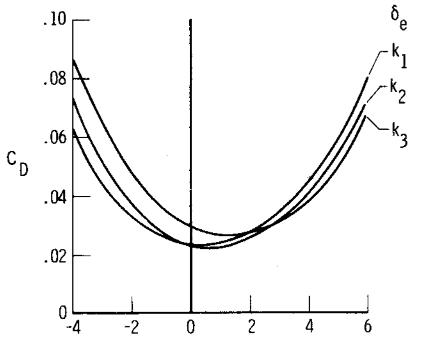

Fig. 4 Wind-tunnel estimates of coefficient of drag as a function of angle of attack for constant elevator deflection.

satisfactory maximum likelihood estimates were obtained accounted for 33% of the total maneuvers. This percentage is somewhat higher than the average failure rate for determining stability and control derivatives with the maximum likelihood estimation method in Ref. 3. The failure rate may be higher for many reasons; for example, this data set contains a high percentage of transonic maneuvers, where aerodynamic characteristics are often difficult to obtain. All of the flight data presented henceforth are for the 14 pushover/pullup maneuvers that were successfully analyzed.

Model Verification

The validity of the model defined by Eqs. (23) and (24) was assessed by comparing estimates of lift and drag obtained with it with estimates obtained from the wind tunnel. Curves of C_D vs α for constant values of δ_e were selected from the wind-tunnel estimates (Fig. 4), and a weighted least-squares

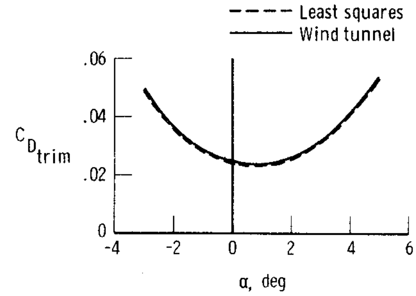

Fig. 5 Comparison between wind-tunnel and assumed model estimates of trimmed drag coefficients.

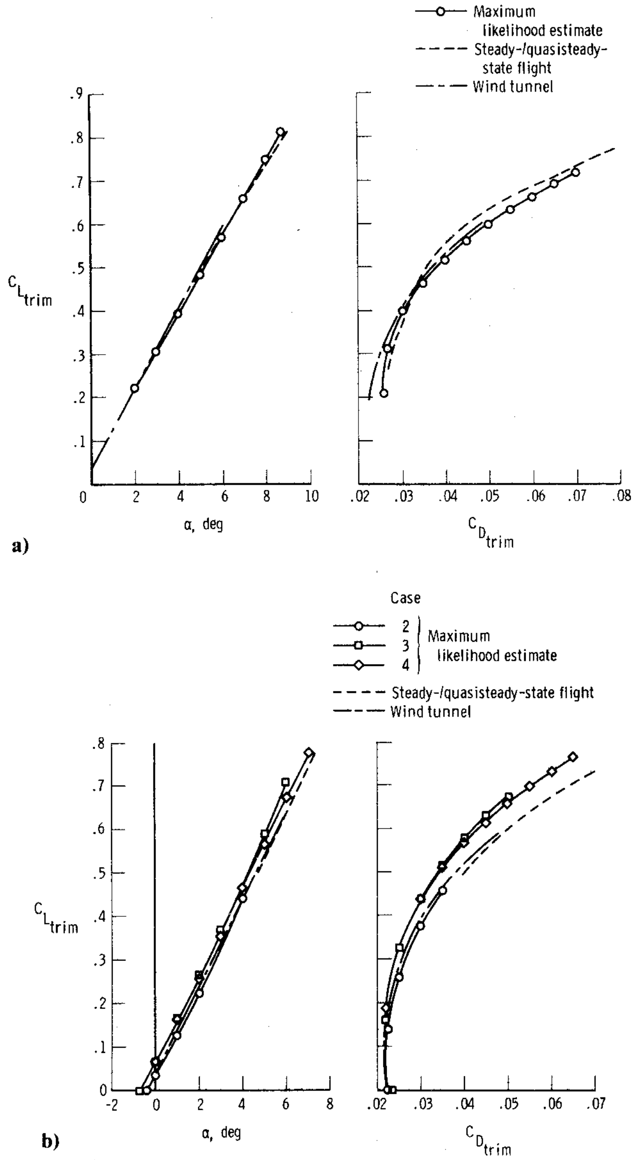

Fig. 6 Coefficients of lift and drag as a function of angle of attack for estimates from the wind tunnel and from flight data analysis methods: a) case 1; b) cases 2, 3, and 4.

fit of these values, based on the power series in Eq. (9), was obtained. Since the flight data were acquired near the trim condition, the wind-tunnel data were weighted to fit near trim. The trimmed wind-tunnel data based on the wind-tunnel data in Fig. 4 are compared with the least-squares fit in Fig. 5. It is apparent that the least-squares fit based on the power series in Eq. (9) agrees closely with the trimmed wind-tunnel estimates of C_D. Similar agreement was obtained for the power-series fit of C_L with the wind-tunnel estimates. The validity of the model was also tested by assessing the quality of the fits obtained (for example, those shown in Figs. 2 and 3). The fits indicated that the model was sufficiently complex for flight data.

Because of the comparison in Figs. 2, 3, and 5, the model defined by Eqs. (23) and (24) was believed to represent the wind-tunnel and flight data accurately. It should be pointed out, however, that the wind-tunnel data were used only to assess the model; no wind-tunnel information was used in the subsequent maximum likelihood estimation analysis of the flight data.

Effect of Thrust Accuracy

To determine aircraft lift and drag characteristics from flight data by any method, an independent estimate of thrust is necessary. The value of the aerodynamic force parallel to the thrust axis is completely dependent on the thrust estimate and can therefore be estimated only within the accuracy of the estimate of the thrust. The thrust for the 14 pushover/pullup maneuvers was determined by using the method described in Ref. 1. The data in Ref. 5 indicate that the accuracy of the thrust determined by using the Ref. 1 method is as good as ±5% at a Mach number of 0.8 and an altitude of 12,000 m. Therefore, the accuracy of the estimate of the drag force is no better than ±5%. This points out the advantage of determining excess thrust instead of drag in that an estimate of thrust is not necessary.

Lift and Drag Characteristics

The aircraft lift and drag characteristics were determined from the 14 pushover/pullup maneuvers. The coefficients extracted were in the body-axis system described by Eqs. (23) and (24), for which the independently measured thrust estimates were included as known variables. These coefficients were then converted to trimmed lift and drag coefficients, C_L and C_D.

Trimmed Lift and Drag Characteristics

To obtain trimmed lift and drag values, lift and drag can be expressed explicitly in power-series form [Eqs. (10-13)] for the angles of attack and elevator deflections that result in zero pitching moment [obtained by equating Eq. (17) to zero].

Figure 6 compares the estimates of $C_{L_{trim}}$ vs α and $C_{L_{trim}}$ vs $C_{D_{trim}}$ obtained from maximum likelihood estimation, steady-/quasisteady-state analysis, and the wind tunnel data obtained at two Mach numbers. Similar plots for all 14 maneuvers are given in Ref. 2. The maximum likelihood estimates are available in curve form, but they are shown as symbols to make it easier to identify estimates obtained from two or more maneuvers made at the same flight condition. Most of the flight data obtained for this vehicle below a lift coefficient of 0.4 were acquired from the pushover/pullup maneuvers during which there was too much pitching motion for adequate analysis by the steady-/quasisteady-state technique. The pilot tried to prevent the pitching motion but was unsuccessful because of the dynamics of the vehicle.

Trimmed Drag Variation with Mach Number

To make a final assessment of the usefulness of the maximum likelihood estimation technique for obtaining drag information from dynamic maneuvers, the maximum likelihood estimates were compared with the steady-/quasisteady-state estimates and the wind-tunnel estimates as a function of the drag-rise Mach number ratio at four constant lift coefficients. Drag-rise Mach number ratio M_r is calculated by dividing Mach number by the wind-tunnel estimate of the drag-rise Mach number at a lift coefficient of 0.25. The drag-rise Mach number is defined as the Mach number where $(\partial C_D/\partial \text{Mach number}) = 0.1$. The maximum likelihood estimates are compared with the steady-/quasisteady-state results in Fig. 7. The dashed line is a fairing of the maximum likelihood estimates similar to those

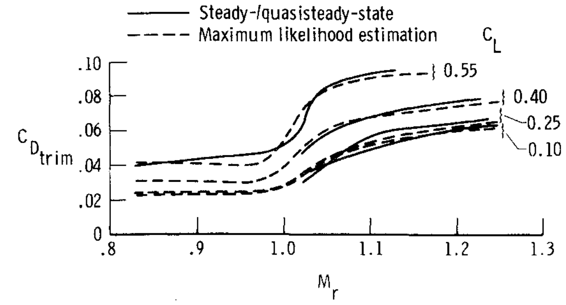

Fig. 7 Coefficient of drag as a function of M_r at constant lift coefficients for estimates from flight-determined methods.

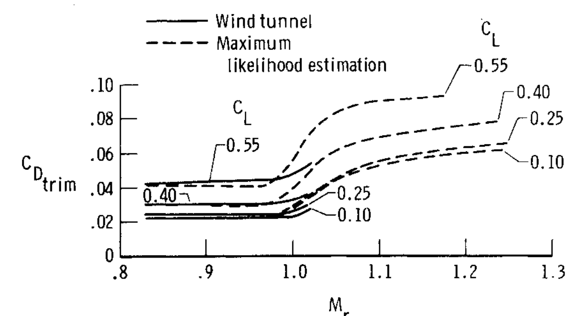

Fig. 8 Summary of coefficient of drag as a function of M_r for estimates from the wind tunnel and the maximum likelihood estimation technique.

presented in Fig. 6. For lift coefficients of 0.10, 0.25, and 0.40, steady-/quasisteady-state estimates are not available for drag-rise Mach number ratios below 1.02, but the maximum likelihood estimates are in good agreement with the steady-/quasisteady-state values at drag-rise Mach number ratios greater than 1.02. A comparison between the maximum likelihood estimation and steady-/quasisteady-state estimates for a lift coefficient of 0.55 throughout the M_r range shows that the maximum likelihood estimates are usually somewhat smaller than the steady-/quasisteady-state estimates.

Figure 8 summarizes the fairings of the constant-lift-coefficient curves as a function of M_r for the maximum likelihood and wind-tunnel estimates. In general, the comparison is good, and indicates that the drag-rise Mach number is somewhat lower according to the maximum likelihood estimation technique than according to the wind-tunnel estimates.

The maximum likelihood estimation method shows promise, and the results may improve with more rapid pushover/pullup maneuvers. Further investigation is needed to determine the best type of maneuver and the best mathematical model to describe the system before a final assessment of the technique can be made.

Concluding Remarks

A maximum likelihood estimation technique was used to derive aircraft lift and drag characteristics from data acquired during dynamic flight maneuvers. The technique tolerates a greater amount of pitching motion than the steady-state and quasisteady-state techniques currently being used to extract lift and drag characteristics from flight data.

In general, the lift and drag coefficients extracted from dynamic flight maneuvers by the maximum likelihood estimation technique agree well with the wind-tunnel estimates. When maneuvers that met the requirements of both flight methods were analyzed, the results of each method were nearly the same.

The maximum likelihood estimation technique was found to show promise in estimating lift and drag characteristics from dynamic flight maneuvers, and further studies should be made to assess the best mathematical model and the most desirable type of dynamic maneuvers to get the highest quality results from the technique.

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